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THE FIGURE OF THE EARTH AND THE MOTION OF THE MOON.

By Prof. Asaph Hall, Washington, D. C.

Having recently examined the history of this question, and finding no connected account of the discovery of the inequalities in the motion of the moon depending on the figure of the earth, the following notes may be of some interest: —

1. The inequality of the longitude of the moon arising from the figure of the earth was found first by Tobias Mayer of Göttingen about 1753. It appears that Mayer discovered this term from the discussion of his own observations of the moon. He made the value of the co-efficient 4″; but as he gave no theoretical reason for the inequality, and as the co-efficient was small with respect to the accuracy of the observations of those times, most of the mathematicians neglected it. But the lunar tables constructed by Mayer were compared with the Greenwich observations by Bradley, 1756 to 1760, who found these tables far superior to any others that had been made. Mayer died in 1762 at the age of thirty-nine, and the English government gave to his widow the sum of three thousand pounds sterling for his lunar tables. It is remarkable, I think, that Mayer was able to detect so small an inequality from his observations of the moon, which could not have been very extensive; but he seems to have been an astronomer who combined sound mathematical knowledge with great skill and judgment in the discussion of astronomical and physical questions.

2. In 1780, Mayer's tables were compared with observations by Charles Mason, who had been an assistant of Bradley, and a new edition was published in 1787. The co-efficients were corrected, and that for the inequality in the longitude which depends on the figure of the earth was found to be 7″.7.

3. About 1800 Bürg of Vienna found by comparing a large number of observations the value of this co-efficient to be 6″.8. Up to this time, no direct theoretical investigation of this question seems to have been made, although, as will be noticed, the period includes the time when Legendre and Laplace were making their celebrated researches on the attraction of ellipsoids.

a. The first theoretical investigation of this question that I have found was by Lagrange in 1773 in his prize memoir of the Paris Academy of 1774. The memoir of Lagrange is on the secular equation of the moon, and he examines the action of the figure of the earth in this equation, and shows that there is no term arising from this source that would produce a secular effect. The investigation of Lagrange is an elaborate one, and covers nearly fifty pages in the sixth volume of his works (edition of Serret). As the purpose was to find only secular terms, the periodical ones were not computed. Lagrange had already con sidered a similar question in his prize memoir on the inequalities of the satellites of Jupiter, 1766, where he gives the expressions of the forces arising from the figure of Jupiter.

β. In 1800, at the request of Bürg, as it appears, Laplace first computed the co-efficients of the principal inequalities produced by the figure of the earth in the longitude and latitude of the moon. The memoir was read "*le 26 prairial an 8*," and is printed in the third volume of the memoirs of the Institute, p. 198. It covers only nine pages, and has been reproduced in the *Mécanique Céleste*, Vol. III. With an assumed figure of the earth, Laplace finds the co-efficient in longitude 5″.6, and in latitude — 6″.5. He points out that in case of the homogeneity of the earth these co-efficients would be much larger, and as this is contradicted by observation, the earth cannot be homogeneous. It appears from this memoir that Laplace had previously made an erroneous calculation, in which he found the co-efficients of these inequalities less than 2″.

γ. It may be worth while to mention in this connection two of the recent determinations of these co-efficients. In his theory of the moon, 1864, Hansen with a new value for the figure of the earth finds the value of the co-efficients 7″.40 and — 8″.38. Mr. G. W. Hill, in a recent paper from the office of the American Ephemeris and Nautical Almanac, 1884, has made an elaborate investigation of this question, and finds for the principal terms 7″.67 and — 8″.73.

These inequalities are interesting, since, combined with the figure of the earth determined from geodetic measurements, they give us some idea of the constitution of our planet. Speaking of these terms, Laplace says: "Les deux inégalités précédentes méritent toute l'attention des observateurs; elles ont sur les mesures géodésiques l'avantage de donner l'aplatissement de la terre d'une manière moins dépendante des irregularités de sa figure."

There is another interesting inequality in the moon's motion, the parallactic equation, which depends on the angular distance of the sun and moon, and which furnishes a means of determining the sun's distance from the earth, or the solar parallax. Tobias Mayer seems to have been the first astronomer who used this inequality for determining the sun's distance. He was also one of the first astronomers to employ equations of condition in the discussion of observations.

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